Acoustic Inhomogeneity in Magnetoacoustic tomography with Magnetic Induction based on Split Bregman Methods

Shuai Zhang*, Wenlong Li*, Hongshuang Yang, Manling Ge, Changgeng Zhang, and Guizhi Xu, Weili Yan

Province-Ministry Joint Key Laboratory of Electromagnetic Field and Electrical Apparatus Reliability, Hebei University of Technology, Tianjin 300130 CHINA, *zs@hebut.edu.cn

Magnetoacoustic tomography with Magnetic Induction (MAT-MI) is a noninvasive electrical conductivity imaging approach that measures ultrasound wave induced by magnetic stimulation, for reconstructing the distribution of acoustic source in biological tissue. Acoustic inhomogeneity significantly affects the propagation of sound waves. The difference in sound velocity results in distortion of the sound source during reconstruction. In order to achieve more computational accuracy especially on the conductivity boundaries and interfaces of inclusions, we provided a new algorithm for MAT-MI to image the impedance distribution in tissues with inhomogeneous acoustic speed distributions. The purpose of this algorithm is to reconstruct the distribution of acoustic source in acoustic inhomogeneous medium more accurately, the reconstruction algorithm is the split Bregman method. In this paper, we built a model of acoustic inhomogeneity and calculated the forward solution, solved the inverse problem. In the forward solution, it is possible to achieve good accuracy and stability using our computational model using Generalized Finite Element Method (GFEM). In the inverse problem, it uses the new algorithm to accurately reconstruct the distribution of sound sources, and the sound speed distribution was reconstructed using symmetrical transducers with the split Bregman method. The results show the feasibility of the forward solver and inverse solver in MAT-MI, and the contrast of reconstruction images could be improved.

***Index Terms*—MAT-MI, split Bregman method, the forward solution, the inverse problem**

INTRODUCTION

Medical imaging technology in modern medicine play an increasingly important role. Different tissues have different conductivities and different electric constants, and their electrical properties are sensitive to both physiological and pathological conditions. Therefore, the imaging method based on the electrical properties of biological tissue has great potential for early diagnosis.

Magnetoacoustic tomography with magnetic induction (MAT-MI) [1] is a hybrid imaging technology combining the ultrasonic and the electrical impedance ones. In MAT-MI, an object is placed in an external static magnetic field B_0 and a time-variant magnetic field B_1 to induce the eddy currents *J* in the object. The eddy currents are subject to Lorentz forces to induce sound vibrations in the object. The emitted sound signals are detected around the object to reconstruct the electrical impedance images of the imaging object.

Existing reconstruction algorithms for MAT-MI are based on the assumption that the acoustic properties in the tissue are homogeneous, the velocity of sound in the tissue was constant and that reflection and refraction were also ignored during propagation of the sound wave. In fact, the speed of sound in human tissue can vary. This change reduces the spatial resolution of the MAT-MI image.

In this study, we used the data detected by the sensor to reconstruct the distribution of acoustic properties in the biological tissue. The speed of sound is used to reconstruct its source. we built a model of acoustic inhomogeneity with GFEM and calculated the acoustic source, solved the distribution of the acoustic pressure signal, and used new algorithm to calculate the sound source reconstruction. Finally, we are able to solve complex sound propagation problems and reconstruct images of sound sources with variable sound speeds.

METHODS

In this paper, we use GFEM to model the inhomogeneity of the tissue and solve inhomogeneity medium. The wave equation governing the pressure distribution is given in (1)

$$\nabla^2 p(r,t) - \frac{1}{c_s^2}\frac{\partial^2 p(r,t)}{\partial t^2} = \nabla \cdot (J \times B_0) \tag{1}$$

where p , r , t and c_s represent the pressure, the spatial point, time and the acoustic speed in acoustic media, respectively.

We use split Bregman method to model the inhomogeneity of the tissue and solve inhomogeneity medium, Conventional algorithms include artificial time evolution (ATM), lagged fixed point algorithm, prime-dual method, and so on. Although these methods are computationally simple, but because of the convergence of the diffusion operator is poor, so these methods are slow convergence, reconstruction is not ideal.

In order to improve the accuracy and convergence of the reconstructed image, a new split Bregman algorithm [2-5] is used to reconstruct the MAT-MI sound source.

First, an auxiliary variable d and an equality constraint are introduced to transform the formula-norm regularization method into a constrained optimization problem

$$\min_{\delta\sigma}\{\frac{1}{2}\left\|J\delta\sigma - \hat{U}\right\|_2^2 + \alpha\left\|d\right\|_1\} \quad s.t.\ d = \delta\sigma \tag{2}$$

And then (2) rewritten into the corresponding unconstrained optimization problem

$$\min_{\delta\sigma,d}\{\frac{1}{2}\left\|J\delta\sigma - \hat{U}\right\|_2^2 + \alpha\left\|d\right\|_1 + \frac{\beta}{2}\left\|d - \delta\sigma\right\|_2^2\} \tag{3}$$

Where greater than zero is called the relaxation factor.

The following note $E(\delta\sigma, d) = \frac{1}{2}\left\|J\delta\sigma - \hat{U}\right\|_2^2 + \alpha\left\|d\right\|_1$, the Bregman distance is defined as

$$D_E^p(\delta\sigma,d,\delta\sigma^k,d^k)=E(\delta\sigma,d)-E(\delta\sigma^k,d^k) -\left\langle p_{\delta\sigma}^k,\delta\sigma-\delta\sigma^k\right\rangle-\left\langle p_d^k,d-d^k\right\rangle \tag{4}$$

among them $p_{\delta\sigma}^k\in\partial_{\delta\sigma}E(\delta\sigma^k,d^k)$, $p_d^k\in\partial_d E(\delta\sigma^k,d^k)$ Respectively, $E(\delta\sigma,d)$ at $(\delta\sigma^k,d^k)$ for the sub-differential of $\delta\sigma$ and d.

It can be seen that the Bregman iteration is a process of converting the original problem l^1 - norm regularization method into an iterative solution of a class of unconstrained optimization problems and updating the Bregman parameter b_d^k .

SIMULATION STUDY AND RESULTS

In the simulation, a cross section of a 3D model with concentric spherical model was used to test the feasibility of the proposed algorithm in solving the forward and inverse problems described above. The parameters of the concentric spherical simulation model are listed in Table 1. The distribution of the acoustic source, the acoustic pressure signal, and the acoustic source reconstruction signal are solved, shown in Fig. 1 to Fig. 6.

TABLE 1

PARAMETERS AND COORDINATES OF THE MODELS

Model Name	Conduct-ivity (S/m)	Radius (m)	Acoustic speed (m/s)
Outer Sphere σ_1	0.25	0.03	1500
Inner Sphere σ_2	0.04	0.01	1950

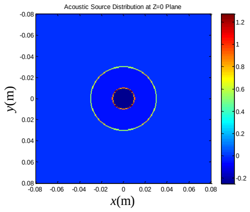

Fig. 1. Acoustic source distribution[mP]

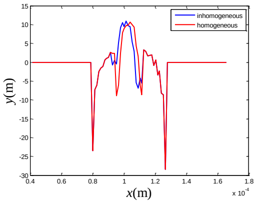

Fig. 2. Comparation of pressures

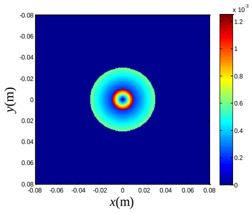

Fig. 3. Eddy current distribution[A/m2]

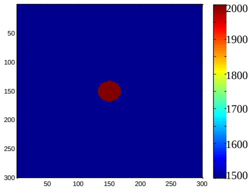

Fig. 4. acoustic speed distribution

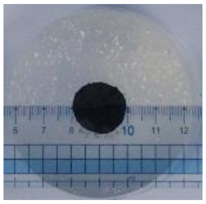

Fig.5. Phantom model

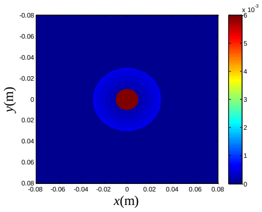

Fig.6. Conductivity to be reconstructed

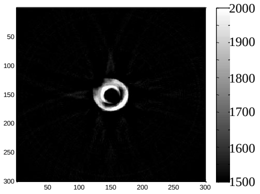

Fig. 7. Reconstructed acoustic speed

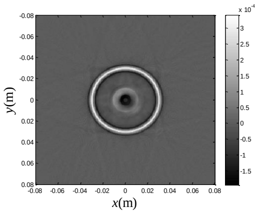

Fig. 8. Acoustic source reconstruction

Fig. 1 the calculated of acoustic source distribution at Z=0 plane was showed. From fig. 2 we can see the distribution of the “acoustic pressure” signal. Fig. 3 shows eddy current distribution. the acoustic speed distribution results are shown in Fig. 4. The phantom model is shown in Fig. 5. Fig. 6 shows the relative conductivity of the actual value, that is, conductivity contrast is restored. The acoustic speed reconstruction results are shown in Fig. 7. Fig. 8 shows acoustic source reconstruction results. Using the algorithms described in section 2.

As shown in Fig. 8, the 2-D MAT-MI image agrees well with the cross section of the phantom in terms of shape and size of the reconstruction. The results show that the phantom model and simulation are identical within tolerance.

CONCLUSION AND DISCUSSION

In conclusion, we combined the relationship between symmetrical transducers with the purpose new method to determine the distribution of sound speed. The processing of acoustic source reconstruction is split-bregman method. The results show that the split-bregman iterative algorithm has a competitive edge in the estimation of the conductivity, reconstructing the image, the position of the edge and the computational speed. Compared with the previous algorithm, this approach accelerates the reconstruction by 50%. Using newly provided algorithm, we have evaluated this approach with an eccentric model, and the feasibility has been tested in our computer simulation studies. The simulation results are promising and suggest that this algorithm is potential to become an important reconstruction approach of MAT-MI.

ACKNOWLEDGMENT

This work was supported in part by the NSF of Hebei Province under Grant No. E2015202292 and E2015202050, research project in Hebei Province under Grant No. C2015005012, No. 15272002 and No. 15275704.

REFERENCES

[1] Xu Y. and He B. “Magnetoacoustic Tomography with Magnetic Induction (MAT-MI)”, *Physics in Medicine and Biology*, vol. 50, 2005, pp. 5175-5187.

[2] L. Proekt and I. Tsukerman, “Method of overlapping patches for electromagnetic computation,” *IEEE Trans. Magn.*, vol. 38, 2002, pp. 741–744.

[3] I. Tsukerman, “General tangentially continuous vector elements,” *IEEE Trans. Magn.*, vol. 39, 2003, pp. 1215–1218.

[4] Tsukerman I., “A flexible local approximation method for electro- and magnetostatics”, *IEEE Trans. Magn.*, vol. 40, no. 2, 2004, pp. 941-944.

[5] Goldstein T, Osher S. The split Bregman method for L1-regularized problems[J]. *SIAM journal on imaging sciences*, 2009, pp: 323-343.